Set, Relation and Function

Set

A well-defined collection of distinct objects is called a set.

Remarks: (i) Every object, included in the set is called an element of the set.

(ii) By well-defined collection of objects we mean that "if A is a set and 'a' is any element then either 'a' is an element of A (denoted by $a \in A$) definitely or 'a' is not an element of A (denoted by $a \notin A$) definitely.

(iii) Usually capital letters of English alphabet are used to denote sets and small letters are used to denote elements of sets.

Example: $A = \{ a, b, c, d\}$ is a set consisting of four elements a, b, c and d.

Standard Numerical Sets

(i) N = The set of natural numbers:

$$N = \{ 1, 2, 3, 4, \dots \}$$

(ii) Z = The set of integers:

$$Z = \{ \dots -3, -2, -1, 0, 1, 2, 3, \dots \}$$

(iii) Z^+ = The positive integers:

$$Z^+ = \{1, 2, 3, 4, 5, \dots \}$$

(iv) Q = The set of rational numbers

$$Q = \{\frac{p}{q} : p, q \in Z \text{ and } q \neq 0\}$$

(v) Q^c = The set of irrational number = A number that can be expressed as an infinite decimal with no set of consecutive digits repeating itself indefinitely and that cannot be expressed as the quotient of two integers

(vi) R = The set of real numbers = The union of set of rational and irrational number

(vii) C = The set of complex number = $\{ x + iy : x, y \in R \}$

Methods of describing a set:

There are two methods of describing of a set.

(a) Roster Method or Tabular Method:

In this method, *A* set is described by listing all its elements, separating then commas and enclosing them with in curly brackets.

Example: (i) If *A* is the set of odd natural number less than 10 then in roster form

$$A = \{ 1, 3, 5, 7, 9 \}$$

(i) If B is the set of letters of the word FOLLOW then in roster form

$$B = \{ F, O, L, W \}$$

(ii) If C is the set of all postive factors of 24 then in roster form

$$C = \{ 1, 2, 3, 4, 6, 8, 12, 24 \}$$

(b) Set Builder Method: Listing the elements of a set in sometimes difficult and sometimes impossible. We do not have a roster form of the set Q of rational number or the set of real number. In set builder method; a set is described by means of some property which is satisfied by all the elements of the set.

i.e. If a set A is defined by a property P, we write

$$A = \{x : P(x)\}$$

and read A is the set of all x such that x has property P.

Example (i): The set $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ in set builder form

$$A = \{x : x \in N \text{ } and \text{ } x < 9\}$$

(ii) If B is the set of all natural number between 10 and 100 then

$$B = \{x : x \in N \ and\ 10 < x < 100\}$$

Difference Types of Sets

(i) Null set or Empty Set or Void set: The set which contain no element is called the empty set. The empty set is denoted by

{ } or $\emptyset$.

Example (i):

$$A = \{x \in Z : x^2 = 2\}$$

$$A = \emptyset$$

(ii)

$$B = \{x \in R : x^2 + 1 = 0\}$$

$$B = \emptyset$$

(ii) Singleton set: The set which contain only one element is called the singleton set.

Example (i):

$$A = \{x \in R : x + 5 = 8\}$$

$$A = \{3\}$$

(ii)

$$B = \{x \in N : 6 < x < 8\}$$

$$B = \{7\}$$

(iii) Finite set: The set which contain finite number of elements is called the finite set.

Example (i):

$$A = \{x \in N : 1 < x < 7\}$$

$$A = \{2, 3, 4, 5, 6\}$$

(iv) Infinite set: The set which contain infinite number of elements is called the infinite set.

Example: N, Z, Q, R, C

Subset

Let A and B be two sets. If every elements of A is also an elements of B then A is called a subset of B. It is denoted by the symbol $A \subseteq B$ and is read as 'A subset B'.

i.e. $\forall x; x \in A \ \textit{implies that} \ x \in B \ \textit{then} \ A \subseteq B.$

Example: If $A = \{2, 4, 6\}$ *and* $B = \{2, 4, 6, 8\}$

Then $A \subseteq B.$

Proper Subset

Let A and B be two sets. Then a subset A of B is said to be a proper subset if A is not equal to B. It is denoted by $A \subset B$.

Notationally: $A \subset B \quad iff \quad \exists\, t \in B \ \textit{such that} \ t \notin A$

Example: If $A = \{ 2, 4, 6\}$ *and* $B = \{ 2, 4, 6, 8\}$

Then $A \subset B$.

Question 1: Which of the following are true:

1. $N \subset R$
2. $Z \subseteq N$
3. $\{-3\} \subseteq \mathrm{R}$
4. $\{-1,2\} \subseteq \mathrm{Z}^{+}$
5. $\emptyset \subseteq \emptyset$

Equal Sets

Two sets A and B are called equal if

(i) Every element of A is also an element of B *i.e.* $A \subseteq B$

(ii) Every element of B is also an element of A *i.e.* $B \subseteq A$

It is denoted by $A = B$.

Example (i): If

$$A = \{x \in Z : x^2 = -1\}$$
$$B = \{1, -1\}$$

Then $A = B$

(ii): If

$$A = \{2\}$$
$$B = \{p : p \text{ is an even prime number}\}$$

Then $A = B$

Power Set

Let A be a set. The set of all subset of A is called the power set of A. It is denoted by $P(A)$.

$$i.e.\ \ P(A) = \{X : X \subseteq A\}$$

Example: If $A = \{ a, b\}$ Find $P(A)$

$$P(A) = \{ \emptyset, \{a\}, \{b\}, \{a,b\}\}$$

Theorem : If a finite set A has n elements then show that the power set of A has 2^n elements.

Proof: Please try yourself (or Discuss in class)

Cardinality of a Set

The cardinality of a set is the number of distinct elements in the set. $|A|$ denotes the cardinality of A.

Question: Find the cardinality of a set

$$A = \{x \in N : 1 < x < 7\}$$

Solution: We get $A = \{2, 3, 4, 5, 6\}$ then $|A| = 5$

Ordered Pair

An ordered pair is pairs of elements written according to a specified order. The pair *(a, b)* of the numbers *a* and *b* is called ordered pair. First number *'a'* is called first co-ordinate (component) and second number *'b'* is called second co-ordinate (component).

Example: If two coins are tossed once, then any one of the following four ordered pairs appears *(H,H)*, *(H,T)*, *(T,H)*, *(T,T)*.

where *H* and *T* denote head and tail respectively.

Note: (i) Two ordered pairs (a_1, b_1) and (a_2, b_2) are said to be equal if and only if $a_1 = b_1$ and $a_2 = b_2$.

Example: If the ordered pair *(2x-1,-5)* and *(x+1, y)* are equal. Find the values of x and y.

Solution: If $(2x-1,-5) = (x+1, y)$

Then $2x-1= x+1$ and $-5 = y$

We get $x = 2$ and $y = -5$.

Cartesian product of two sets

Let A and B be two sets. The Cartesian product of the sets A and B is the set of all those ordered pairs whose first co-ordinate is an element of A and second co-ordinate is an element of B and is denoted by $A \times B$.

i.e. $A \times B =\{(a, b): a \in A \text{ and } b \in B\}$

Similarly $B \times A =\{(b, a): b \in B \text{ and } a \in A\}$

Example: If $A=\{1, 2, 3\}$, $B = \{2, 3, 4\}$

$A \times B = \{ (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$

$B \times A = \{ (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (4,1), (4,2), (4,3)\}$

We see that $A \times B \neq B \times A.$

Note: (i) If the set A has m elements and set B has n elements then product set $A \times B$ has mn elements.

(ii) In general $A \times B \neq B \times A$

(iii) If $B = A$ then the Cartesian product $A \times A$ is usually denoted by A^2.

i.e. $A^2 = A \times A = \{(a_1, a_2) : a_1, a_2 \in A \}.$

Relation

In order to express a relation from set *A* to the set *B*, we always need a statement which connects the elements of *A* with the elements of *B*.

For example Suppose $A=\{1, 3, 5, 9\}$, $B=\{0, 2, 4, 8\}$. Now suppose a relation from the set *A* to set *B* is expressed by the statement 'is less than'.

Now taking the first and second co-ordinate of the elements of the set *A* and *B* respectively, the ordered pair satisfying the statement 'is less than' are as follows: $(1, 2), (1, 4), (1, 8), (3, 4), (3, 8), (5, 8)$

The set *R* of these ordered pairs given by

$$R = \{ (1, 2), (1, 4), (1, 8), (3, 4), (3, 8), (5, 8) \}$$

Express a relation from the set *A* to the set *B*.

Clearly, the set *R* is a subset of *A* x *B*.

Definition of Relation

Let *A* and *B* be two sets. A relation from *A* to *B* is a subset of *A* x *B* and is denoted by *R*. Symbolically:

R is a relation from A to $B \Leftrightarrow R \subseteq A \times B.$

Clearly, any relation from *A* to *B* is given by the following set:

$$R = \{(x, y): xRy, where\ x \in A\ and\ y \in B\}$$

Total number of Relation

If the set *A* has *m* elements and the set *B* has *n* elements then

the total number of all relation from *A* to *B* = 2^{mn}.

Domain and Range of a Relation

Suppose R is relation from A to B $i.e.$ $R \subseteq A \times B$. The domain of R is the set of all first elements of the ordered pairs which belongs to R and range of R is the set of all second elements of those ordered pairs.

Domain of $R = \{x: x \in A \; and \; (x, y) \in R \; for \; some \; y \in B\}$

Range of $R = \{y: y \in B \; and \; (x, y) \in R \; for \; some \; x \in A\}$

Inverse Relation

Suppose R is relation from A to B then the inverse of R is a relation from B to A and is denoted by R^{-1}. Symbolically

$$R^{-1} = \{ (y, x): (x, y) \in R \,, x \in A, y \in B\}$$

Example 1: If $A = \{a, b\}$ and $B = \{c, d, e\}$ then find the total number of relation from A to B.

Solution: Since, A has 2 elements and B has 3 elements.

Therefore total number of relation from A to $B = 2^6 = 64$.

Example 2: If a relation $R = \{ (x, y): x, y \in N \; and \; x + y = 8\}$ then find the domain, range of R.

Solution: Here, $x, y \in N = \{1, 2, 3, 4, \ldots\}$

The values of x and y satisfying $x + y = 8$ are given in the following table:

$x =$	1	2	3	4	5	6	7
$y =$	7	6	5	4	3	2	1

Hence, $R = \{ (1, 7), (2, 6), (3, 5) (4, 4), (5, 3), (6, 2), (7, 1)\}$

Domain of $R = \{1, 2, 3, 4, 5, 6, 7\}$

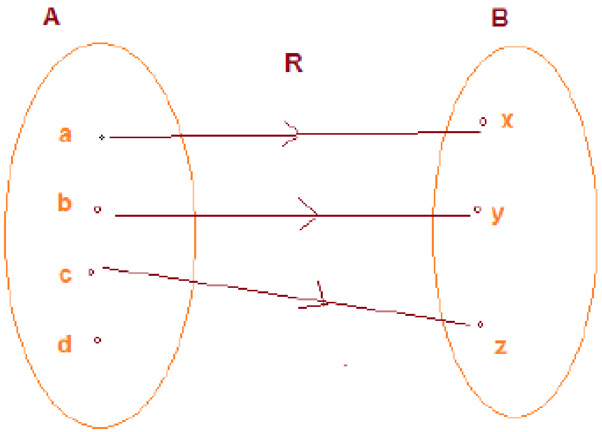
Range of $R = \{1, 2, 3, 4, 5, 6, 7\}$

Inverse $R^{-1} = \{ (1, 7), (2, 6), (3, 5) (4, 4), (5, 3), (6, 2), (7, 1)\}$

Pictorial Representation of Relation

A Relation R from A to B can be depicted pictorially using arrow diagram . In arrow diagram , we write down the elements of two set A and B in two disjoint circle ,Then we draw arrow from set A to set B whenever $(a, b) \in R$

Let A={a,b,c,d} and B={x,y,z}
And R ={(a,x),(b,y),(c,z)}
Then this will be represented in arrow diagram as



Example:

Let $\boldsymbol{P = \{1, 2, 3, \ldots\ldots, 18\}}$ define a relation $\boldsymbol{R}$ from $\boldsymbol{P}$ to $\boldsymbol{P}$ by $\boldsymbol{R = \{(x, y) : 2x - y = 0, where\ x, y \in P\}}$ Write down its domain, co-domain and range.

Draw the arrow diagram for the relation also

Solution: The relation $\boldsymbol{R}$ from $\boldsymbol{P}$ to $\boldsymbol{P}$ is given as

R = {(x,y):2x-y=0, where x, y ∈ P}

i.e., R = {(x, y): 2x = y, where x, y ∈ P}

Therefore,

$\boldsymbol{R = \{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10), (6, 12), (7, 14), (8, 16), (9, 18)\}}$

$\boldsymbol{Domain\ of\ R = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}}$

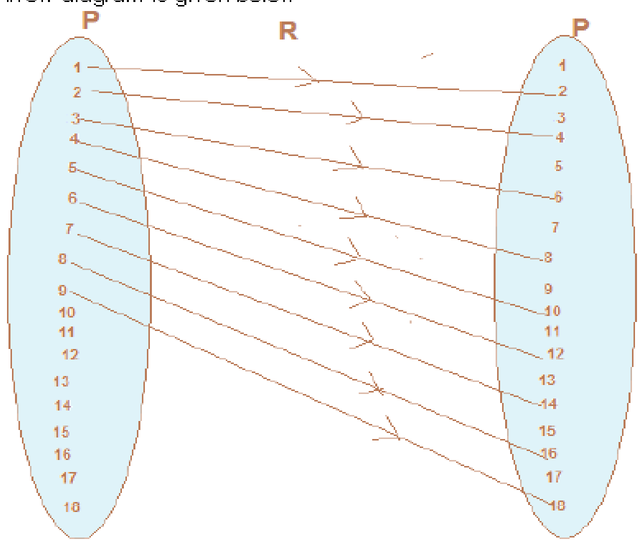
The whole set $\boldsymbol{P}$ is the co-domain of the relation $\boldsymbol{R}$.

Therefore co-domain of $\boldsymbol{R = P = \{1, 2, 3, \ldots, 18\}}$

The range of $\boldsymbol{R}$ is the set of all second elements of the ordered pairs in the relation.

Therefore range of $\boldsymbol{R = \{2, 4, 6, 8, 10, 12, 14, 16, 18\}}$

Arrow diagram is given below



Binary Relation (OR Relation in a Set)

Suppose A is a non-empty set. A relation R in a set A is the subset of $A \times A$. This relation R in the set A is called a binary relation.

i.e.

$$R = \{(x, y): xRy, where\ x, y \in A\}$$

Example: Let $A = \{1, 2\}$

Then $R = \{(1, 1), (1, 2), (2, 2)\}$ is binary relation on A.

Types of Relation

1. **Identity Relation:** Let $R \subseteq A \times A$ be relation on set A. The identity relation defined as

$$I = \{(a, a): \forall a \in A\} \text{ only.}$$

Example: If $A = \{1, 2\}$

and $I = \{ (1, 1), (2, 2)\}$ is relation on A.
Then I is an identity relation on A.

2. **Reflexive Relation:** A relation $R \subseteq A \times A$ is known as reflexive relation if each member of A is related to itself at least.

Example: A = {1, 2, 3} and

$$R_1 = \{ (1, 1), (1, 2), (2, 3)\}$$
$$R_2 = \{ (1, 1), (2, 2), (3, 3),(1, 2)\}$$
$$R_3 = \{ (1, 1), (2, 2), (2, 1),(3,2)\}$$
$$R_4 = \{ (1, 1), (2, 2), (3, 3), (1,2), (3,1), (3,2)\}$$

In above example, R_2 and R_4 are reflexive relation on the set A.

3. **Symmetric Relation:** A relation $R \subseteq A \times A$ is known as symmetric relation

If $aRb \Rightarrow bRa$

i.e. $if \quad (a, b) \in R \Rightarrow (b, a) \in R$

$where\ a, b \in A$

Example: A = {1, 2, 3} and

$$R_1 = \{ (1, 1), (1, 2), (2, 3)\}$$
$$R_2 = \{ (1, 1), (2, 2), (3, 3),(1, 2)\}$$
$$R_3 = \{ (1, 1), (2, 2), (2, 1),(1,2)\}$$
$$R_4 = \{ (3, 3), (3,2), (2,3)\}$$

Here, R_3 and R_4 are symmetric relation on the set A.

4. **Transitive Relation:** A relation $R \subseteq A \times A$ is known as transitive relation

If $$aRb \ and \ bRc \Rightarrow aRc$$

i.e. $$if \quad (a, b) \in R \ and \ (b, c) \in R \Rightarrow (a, c) \in R$$

$where\ a, b, c \in A$

Example: A = {1, 2, 3} and

$$R_1 = \{ (1, 1), (1, 2), (2, 1)\}$$

$$R_2 = \{ (1, 2), (3, 4)\}$$

$$R_3 = \{ (1, 2), (2, 1), (2, 2),(1,1)\}$$

$$R_4 = \{ (2, 3)\}$$

Here, R_2 , R_3 and R_4 are transitive relation on the set A.

Equivalence Relation

A relation R in a set A is said to be equivalence relation if it is reflexive, symmetric and transitive.

Example: A = {1, 2, 3} and

$$R_1 = \{ (1, 1), (2, 2), (3, 3), (1,2), (2, 1),(3,1), (1,3)\}$$

Here, R_1 is an equivalence relation on the set A.

Example:

Let $A = R$ and define the "square" relation

$$R = \{(x,y) \in R \mid x^2 = y^2\}.$$

The square relation is an equivalence relation because

1. For all $x \in R$, $x^2 = x^2$, so $(x,x) \in R$.
2. If $(x,y) \in R$, $x^2 = y^2$, so $y^2 = x^2$ and $(y,x) \in R$.
3. If $(x,y) \in R$ and $(y,z) \in R$ then $x^2 = y^2 = z^2$, so $(x,z) \in R$.

Example:

Show that the relation R is an equivalence relation in the set A = Z given by the relation R = { $(a, b) \in Z : a+b$ is even }.

Solution:

$R = \{ (a, b) \in Z : a+b \text{ is even} \}$. Where $a, b \in Z$.

Reflexive Property:

From the given relation, For all $a \in Z$ such that $a + a = 2a = even$

Therefore, $(a, a) \in$ R

Hence R is Reflexive

Symmetric Property:

From the given relation, If $(a,b) \in R$ such that $a + b = even$

i.e. $b + a = even$

Therefore, if $(a, b) \in R$, then $(b, a) \in R$

Hence R is symmetric

Transitive Property:

From the given relation, If $(a,b) \in R$ and $(b,c) \in R$ such that

$$a + b = even = 2r \quad and \quad b + c = even = 2m$$

then
$$a + b + b + c = 2r + 2m$$
$$a + c = 2\,(r + m - b) = even$$

Therefore, if $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$

Hence R is transitive.

Introduction of function

Consider $X = \{1, 2, 3\}$ and $Y = \{2, 4, 5, 6\}$

Consider the relation $R_1 = \{(1,2), (2,4), (3,5)\}$

$R_2 = \{(1,2), (2,5)\}$

Here, we observe that Domain of $R_1 = \{1,2,3\} = X$

and Domain of $R_2 = \{1,2\}$

In R_1, every element of X is related to one element of Y. Here, relation R_1 is a map from X to Y.

The map generally denotes by f in place of R.

Here, in R_2 , every element in X is not related to element of Y.

Therefore, R_2 is not a map.

Remark: Every map is relation but not conversely.

Function

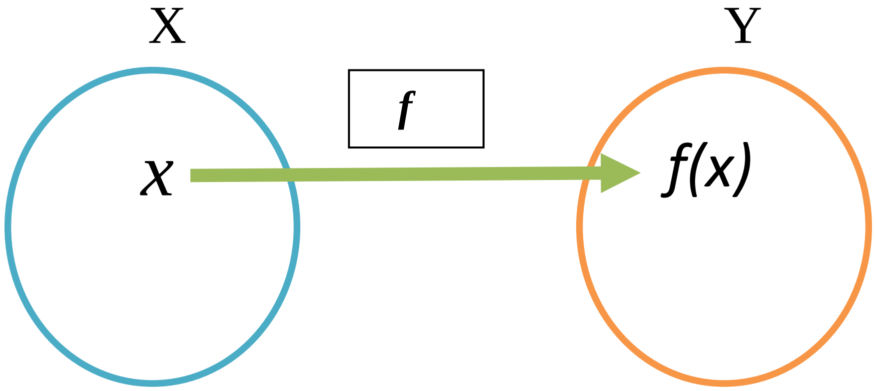
A function is a special type of relation.

<u>Definition:</u> Let X and Y be two non-empty sets. If every element of the first set X is related to an unique element of Y by some definite rule f, then f is called a function or mapping from X to Y and is denoted by

$$f : X \to Y \quad \text{or} \quad X \xrightarrow{f} Y$$

(OR)

A function is a rule that assigns each input exactly one output.



Function Notation

$$y = f(x)$$

Output — **Name of Function** — **Input**

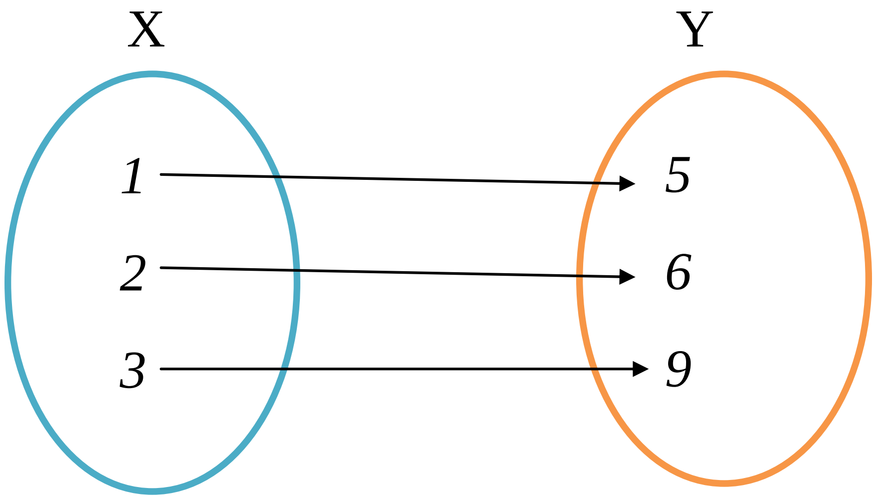
Definition: If $f: X \to Y$ be a map then

Domain of $f = X$

Co-domain of $f = Y$

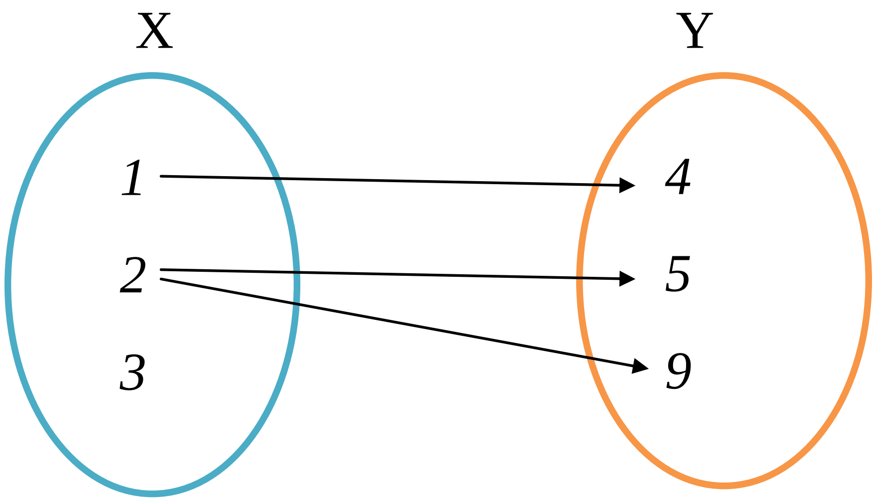
Range $f = \{f(x): x \in X\}$

Example 1:



Domain of $f = \{2,4,5\}$, Co-domain of $f = \{5, 6, 9\}$ and Range $f = \{5, 6, 9\}$.

Example 2:



This is not a map.

Question: Let $X = \{1, 2, 3\}$ and $Y = \{4, 5\}$

Find whether the following subset X x Y are map from X to Y.

(i) $f_1 = \{(1,4), (1,5), (2,4), (3,5)\}$

(ii) $f_2 = \{(1,4), (2,4), (3,4)\}$

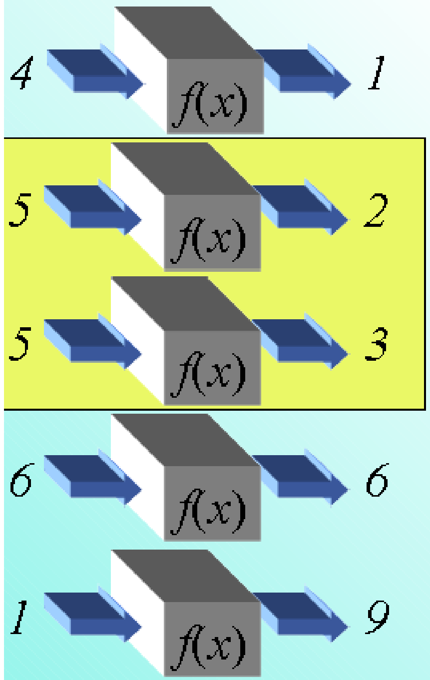
(iii) $f_3 = \{(1,4), (2,5), (3,5)\}$

(iv) $f_4 = \{(1,4), (2,5)\}$

Answer: (ii) and (iii)

Determine whether the relation is a function.

{(4, 1), (5, 2), (5, 3), (6, 6), (1, 9)}



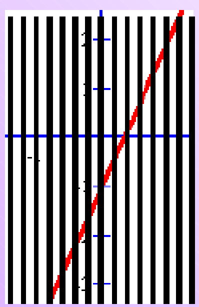
NO,

5 is paired with 2 numbers!

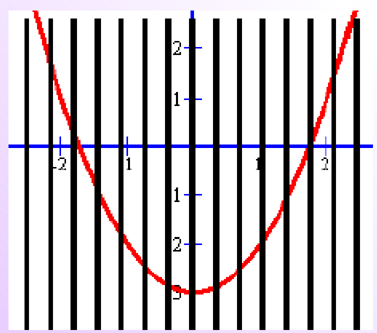
Vertical Line Test (pencil test)

If any vertical line passes through more than one point of the graph, then that relation is not a function.

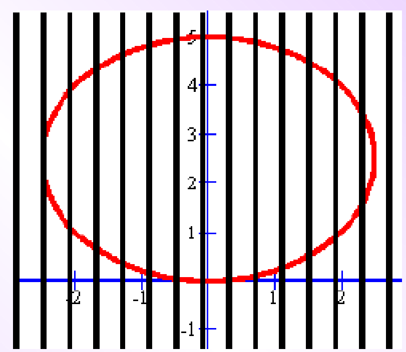
Are these functions?



FUNCTION!

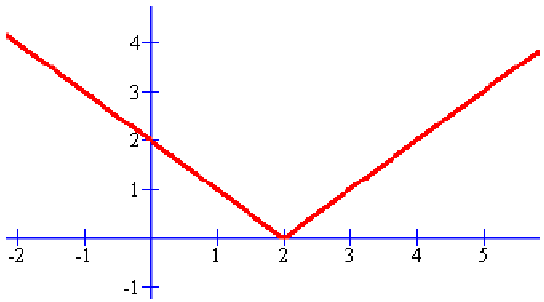


FUNCTION!



NOPE!

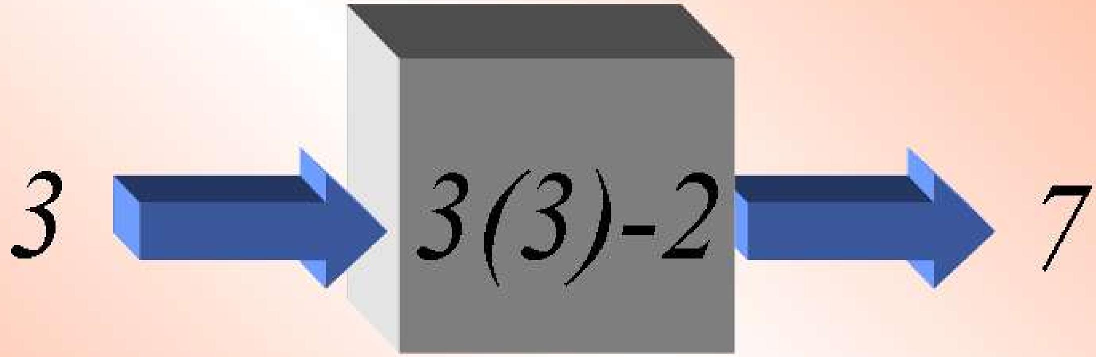
Is this a graph of a function?



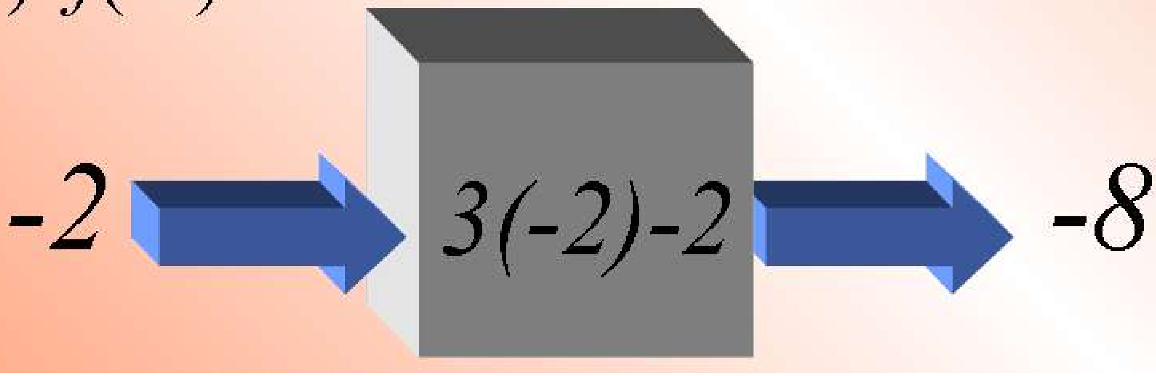
✔ 1. Yes
2. No

Given $f(x) = 3x - 2$, find:

1) $f(3) = 7$



2) $f(-2) = -8$



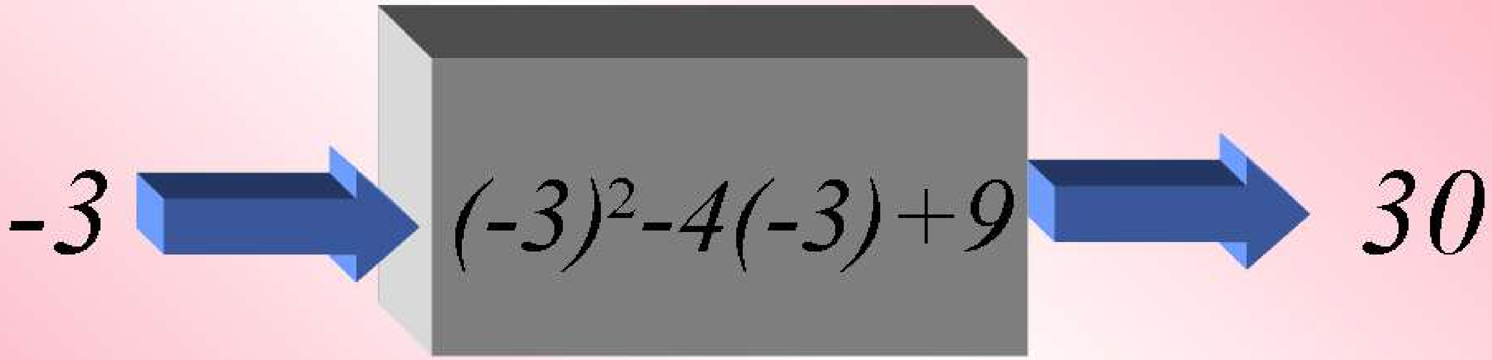
Given $g(x) = x^2 - 2$, find $g(4)$

1. 2
2. 6
3. 14
4. 18

Given $f(x) = 2x + 1$, find $-4[f(3) - f(1)]$

1. -40
2. -16
3. -8
4. 4

Given $h(z) = z^2 - 4z + 9$, find $h(-3)$



$9 + 12 + 9$

$h(-3) = 30$

Number of Maps

Let X be a set containing m elements and Y be a set containing n elements.

Then number of maps = $n^m = (codomain)^{domain}$.

Example: If |X| = 4 and |Y| = 9

Then no. of maps = 9^4 .

Remark: Number of relation > Number of map.

Nature of Maps (Types of Maps)

There are four types of maps

(i) f is one-one.

(ii) f is many-one.

Function is either one-one or many-one

(Output is only one function)

(iii) f is onto.

(iv) f is into.

Function is either onto or into

(Output is only one function)

One - One Map (Injective Map)

A map $f: X \to Y$ is called one-one map if

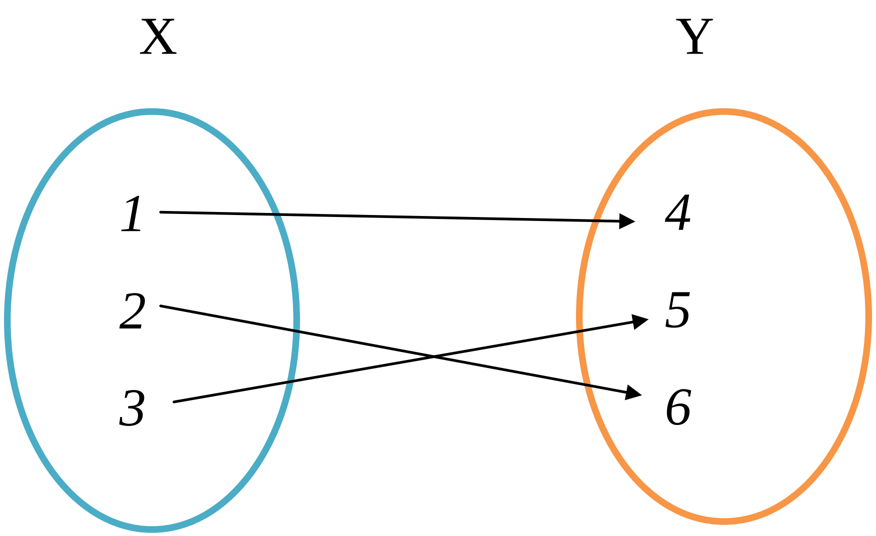
$$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2) \quad where\ x_1, x_2 \in X$$

OR

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \quad where\ x_1, x_2 \in X$$

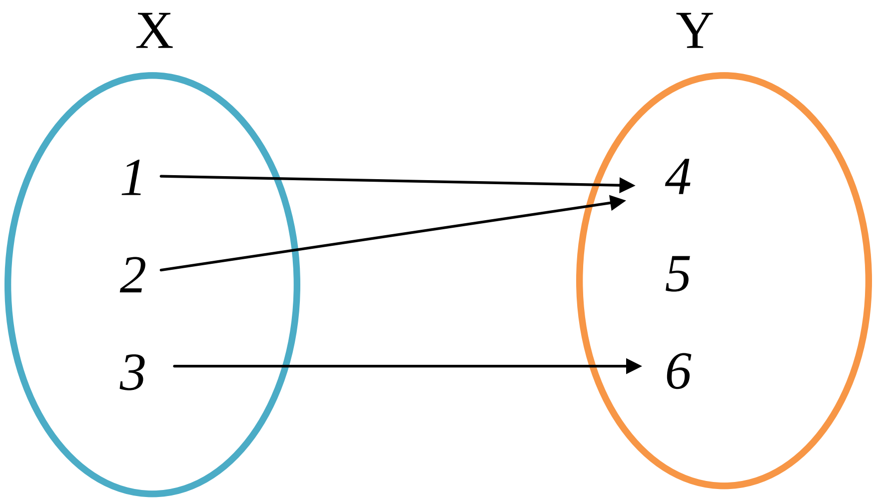
Otherwise it is many one map.

Example 1:



Here, f is one – one.

Example 2:



Here, f is not one – one. (f is many- one.)

Example: Let $f: Z \to Z$ be a mapping such that

(i) $f(x) = x$ (ii) $f(x) = 2x + 3$ (iii) $f(x) = x^2$ (iv) $f(x) = x^2 + 1$

Check f is one-one or not.

Solution: (ii) Let x_1 and x_2 be an arbitrary element of Z.

Then $$f(x_1) = f(x_2)$$

$$2x_1 + 3 = 2x_2 + 3$$

$$x_1 = x_2$$

Hence, f is one-one.

Similarly, (i), (iii) and (iv) : Try yourself.

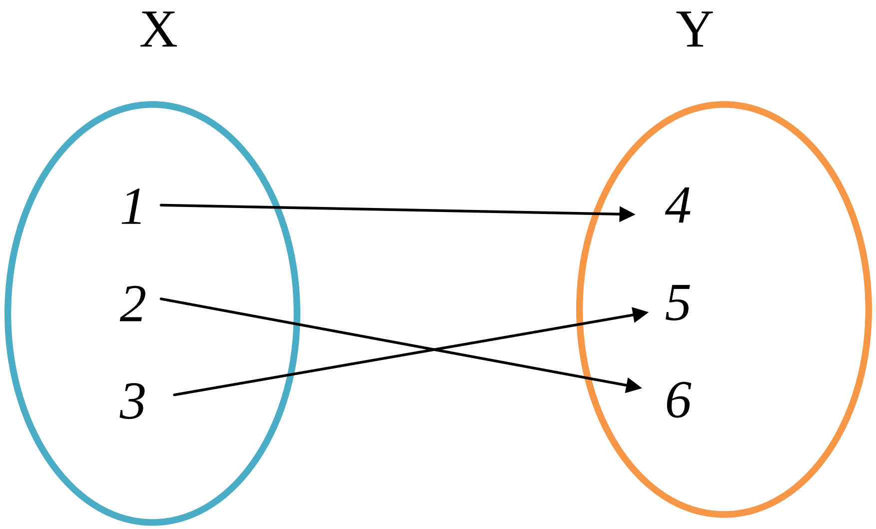
Onto Map (Surjective Map)

A map $f: X \to Y$ is called onto map if $f(x) = Y$

$i.e.$ $y \in Y, \exists\, x \in X \text{ such that } f(x) = Y$

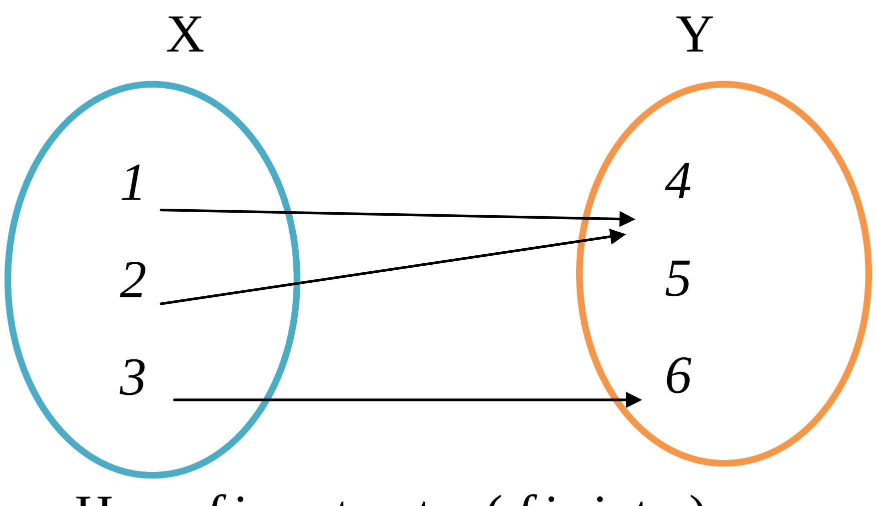
Otherwise it is into.

Example 1:



Here, *f* is onto.

Example 2:



Here, *f* is not onto. (*f* is into.)

Example: Let $f: R \to R$ be a mapping such that

(i) $f(x) = x + 5$

(ii) $f(x) = 2x + 3$

(iii) $f(x) = x^2$

(iv) $f(x) = x^2 + 9$

Check f is onto or not.

Solution: (ii) Let $y \in R$ be an arbitrary element such that

Then $$f(x) = y$$

$$2x + 3 = y$$

$$x = \frac{y-3}{2} \in R \ \ for\ every\ y \in R$$

Hence, f is onto.

Similarly (i), (iii) and (iv) : Try yourself.

Bijective Map

A map $f: X \to Y$ is said to be bijective map if it is one-one and onto.

Example : A map $f: R \to R$ such that f(x) = 2x + 3

Here, f is one – one and onto.

Hence, f is bijective map.

Similar sets (Equivalent Sets)

Two sets are said to be similar if there exist (∃) a one- one and onto map (bijective map) between them.

Remark: Sets are either finite or infinite.

Example : $f: N \to Z$

$$f(n) = (-1)^n \left[\frac{n}{2}\right]$$

$$f(n) = \begin{cases} -\dfrac{n-1}{2} & ; \quad n \text{ is odd} \\ \dfrac{n}{2} & ; \quad n \text{ is even} \end{cases}$$

Here, *f* is one – one and onto.

Hence, $N \sim Z$.

Countably Infinite Set

A set is said to be countably infinite set it is similar to the set of natural number. In that case A is countably infinite then it is always expressed as

$$A = \{a_1, a_2, a_3, \dots, a_n\}$$

Example: N, Z are countably infinite set.

Countable Set

A set is said to be countable set if it is either finite or countably infinite.

Example: N, Z are countable set.

Uncountable Set

A set is said to be uncountable set if it is neither finite nor countably infinite set.

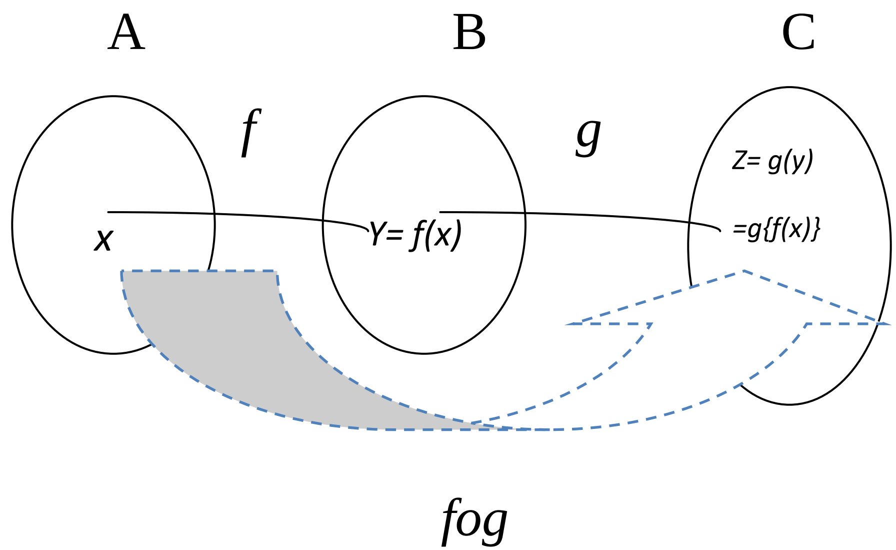
Example: R is uncountable set.

Composite (or Product) of Mapping

Let *A*, *B*, *C* be three non-empty sets. Consider two mappings

$f: A \to B$ and $g: B \to C$ or consider two mapping $f: A \to B$ defined by $y = f(x)$ where $x \in A, y \in B$ and $g: B \to C$ defined by $z = g(y)$ where $y \in B$, $z \in C$ .The composition of mapping *f* and *g* (in this order) is the composite mapping denoted by *g o f* and is a function $f: A \to C$ defined by

$$gof: A \to C \quad \text{where } gof(x) = g\{f(x)\}, \forall x \in A$$



Example: If $f: R \to R$ defined by $f(x) = x^2; \ \forall x \in R$

And $g: R \to R$ defined by $g(x) = \sin x, \ \forall x \in R$

The find gof and fog show that $gof \neq fog$

Solution: Let $x \in R$

$$gof(x) = g\{f(x)\} = g(x^2) = \sin x^2$$

And $$fog(x) = f\{g(x)\} = f(\sin x) = sin^2 x$$

Clearly, $$gof \neq fog.$$

Example: If $f: R \to R$ defined by $f(x) = x^3; \ \forall x \in R$

And $g: R \to R$ defined by $g(x) = \cos x, \ \forall x \in R$

The find gof and fog.

Solution: Please try yourself.